

A METHOD FOR DESIGNING ROBUST MULTIVARIABLE FEEDBACK SYSTEMS

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ABSTRACT

A new methodology is developed for the synthesis of linear, time-invariant (LTI) controllers for multivariable LTI systems. The aim is to achieve stability and performance robustness of the feedback system in the presence of multiple unstructured uncertainty blocks; that is, to satisfy a frequency-domain inequality in terms of the structured singular value.

The design technique is referred to as the Causality Recovery Methodology (CRM). Starting with an initial (nominally) stabilizing compensator, the CRM produces a closed-loop system whose performance-robustness is at least as good as, and hopefully superior to, that of the original design. The robustness improvement is obtained by solving an infinite-dimensional, convex optimization program. A finite-dimensional implementation of the CRM has been developed, and it has been applied to a multivariable design example.

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1. INTRODUCTION

Maintaining stability in the presence of uncertainty has long been recognized as the crucial requirement for a closed-loop feedback system [1, 2]. Classical designers developed the concepts of gain and phase margin to quantify stability-robustness measures. In the modern control era, criteria for maintaining closed-loop stability in the presence of a single, unstructured (i.e. norm-bounded) modeling uncertainty have been formulated in terms of a singular value frequency-domain inequality on the closed-loop transfer function [3].

Recently, the issue of multiple modeling uncertainties appearing at different locations in the feedback loop, and the related requirement of performance-robustness, has been addressed [4]. Multiple unstructured uncertainty blocks, real parameter uncertainty, and performance specifications give rise to so-called structured uncertainty. A new analysis framework, based on the structured singular value μ , has been proposed by Doyle to assess the stability and performance robustness of linear, time-invariant (LTI) feedback systems in the presence of structured uncertainty [5].

While the analysis aspect of LTI feedback design is well-established, the definitive robust synthesis methodology has yet to be developed. The design of a feedback system that exhibits closed-loop stability *and* performance in the face of modeling uncertainty is the so-called " μ -synthesis" problem [6-8]. The synthesis approach proposed by Doyle in [6] is an iterative scheme, referred to as DK iteration, that involves a sequence of scaled H_∞ -based feedback design problems. Unfortunately, convergence to the global solution is not guaranteed due to the inherent nonconvexity of the problem. Since local solutions may result, it is worthwhile to explore other fundamentally different approaches to μ -synthesis that may result in feedback systems with improved robustness properties. In addition, when the CRM was developed the solution of H_∞ problems was computationally very cumbersome; this situation has now changed.

The block diagram in Figure 1.1 has become the standard framework for considering the robust feedback design problem [6-8]. This diagram represents any linear interconnection of inputs, outputs, perturbations, and a compensator. P is the known model that contains the plant to be controlled, and any weighting functions that describe the frequency-domain characteristics of the modeling uncertainty and performance specifications. Δ represents a perturbation due to the modeling error; it is a member of the set $\mathcal{\Delta}$, where

$$\begin{aligned}\mathcal{\Delta} &= \{ \Delta \mid \Delta = \text{diag}(\Delta_1, \Delta_2, \dots, \Delta_n), \Delta_i \in \mathbb{P} \} \\ \mathbb{P} &= \{ \Delta \mid \Delta \text{ stable, } \|\Delta\|_\infty < 1 \}\end{aligned}\tag{1.1}$$

K is the compensator to be designed. The synthesis objective is to find a K to achieve nominal stability and performance of the feedback loop, and to provide robustness with respect to the modeling error. Simply stated, K should be chosen so that the closed-loop transfer function matrix from the exogenous inputs d to the error signals e is "small" for all $\Delta \in \mathcal{\Delta}$. In the sequel, a method is presented for computing such a compensator K .

Section 2 discusses the analysis of the system in Figure 1.1. The structured singular value μ is shown to be an essential tool for dealing with the problem of robust performance. The CRM, a synthesis method based on μ , is presented in Section 3. Section 4 contains a numerical example of a CRM design.

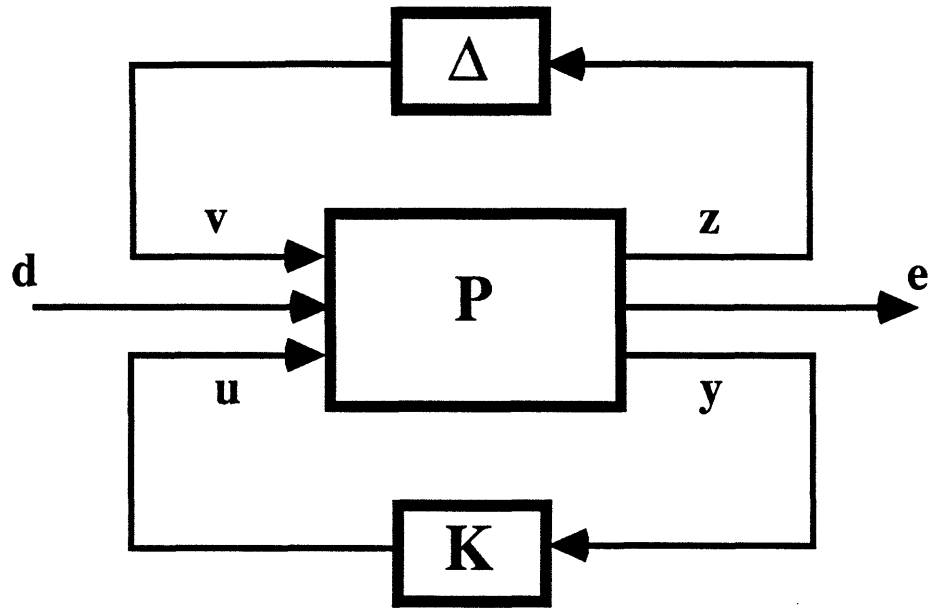


Figure 1.1 General framework for the robust feedback design problem.

2. ANALYSIS

In this section, well-known results pertaining to the stability and robustness analysis of the system in Figure 1.1 are briefly summarized. The compensator K in Figure 1.1 is known for the purposes of analysis, and is incorporated with the plant P via a lower linear fractional transformation to yield the closed-loop operator S (Figure 2.1).

$$S = F_l(P, K) = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21} \quad (2.1)$$

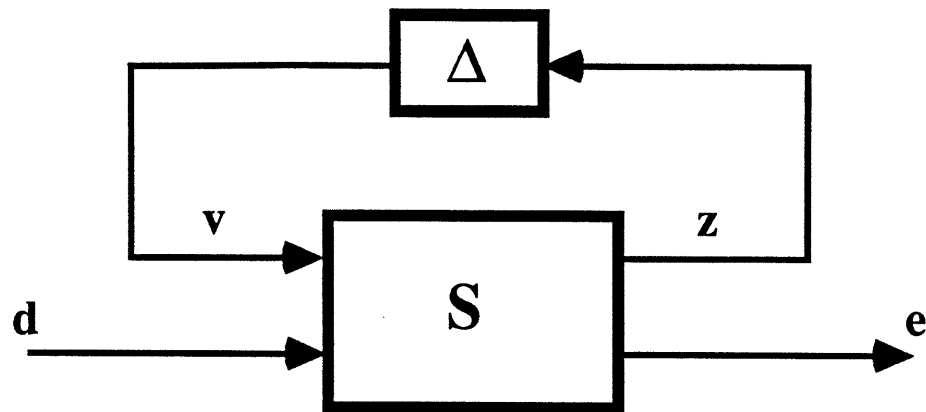


Figure 2.1 Analysis block diagram.

Nominal Stability

For a perturbation Δ identically equal to zero, stability of S will be guaranteed by the Youla parameterization of all internally stabilizing compensators [9, 10]. All such compensators are described in terms of coprime factorizations of the plant P and a free parameter $Q \in H_\infty$. This compensator structure results in an internally stable closed-loop map S that is affine in the free parameter Q , i.e.

$$S = T_{11} + T_{12} Q T_{21} \quad (2.2)$$

where T_{ij} is a function of the plant P and is in H_∞ .

Stability and Performance Robustness

The closed-loop transfer function from the inputs d to errors e in Figure 2.1 is given by the upper linear fractional transformation $F_u(S, \Delta)$.

$$F_u(S, \Delta) = S_{22} + S_{21}\Delta(I - S_{11}\Delta)^{-1}S_{12} \quad (2.3)$$

Then, to satisfy the stability and performance robustness requirement, $F_u(S, \Delta)$ must be stable and "small" *for all possible* $\Delta \in \Delta$. The following theorem establishes the robustness criterion.

Robust Performance Theorem [8]

$F_u(S, \Delta)$ is stable and $\|F_u(S, \Delta)\|_\infty \leq 1 \quad \forall \Delta \in \Delta$ if and only if

$$\|\mu[S(j\omega)]\|_\infty \leq 1$$

where μ is the structured singular value computed with respect to the appropriate block structure.

From the properties of the structured singular value in [5] and Eqn. (2.2), the Robust Performance Theorem is satisfied if

$$\|D(T_{11} + T_{12}QT_{21})D^{-1}\|_\infty \leq 1 \quad (2.4)$$

for some diagonal scaling transfer function D and a $Q \in H_\infty$.

3. SYNTHESIS

The synthesis problem will be discussed with respect to the block diagram in Figure 3.1. The nature and structure of the perturbation Δ impose known constraints on the feedback system; hence, Δ may be ignored for now.

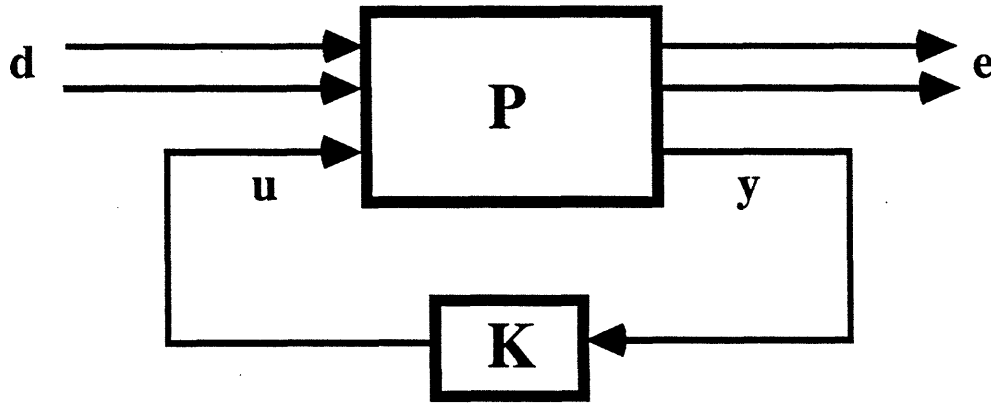


Figure 3.1 Synthesis block diagram.

From the previous section on analysis, we know that a compensator K can be found to meet the design objectives if a function Q exists such that

$$Q \in H_{\infty} \quad (3.1)$$

$$\|D(T_{11} + T_{12}QT_{21})D^{-1}\|_{\infty} \leq \gamma \quad (3.2)$$

At each frequency, D is a known, real, diagonal scaling matrix. Note that γ is just a scale factor to ensure the synthesis problem has a solution. The CRM will find the minimum γ and a transfer function matrix Q that satisfies (3.1) and (3.2), for fixed scaling D . The compensator K in Figure 3.1 is then computed as a function of the Q parameter and the

coprime factorization of P .

The first step of the CRM is the design of a nominally stabilizing compensator K_{nom} for the design plant model P . This may be accomplished by any existing synthesis method; the robustness of this design is a lower bound on the robustness of the feedback loop to be designed by the CRM. The H_∞ methodology [11, 12, 13] provides a reasonable starting design for the CRM since the largest singular value is an upper bound on the structured singular value [5]. The nominal closed-loop map is simply $S_{\text{nom}} = F_1(P, K_{\text{nom}})$.

The robustness properties of S_{nom} are determined by computing an upper bound on $\mu[S_{\text{nom}}(j\omega)]$ [5]. This will result in a real, diagonal scaling matrix D_{nom} at each frequency, and a measure of nominal robustness γ_{nom} .

$$D_{\text{nom}}(\omega) = \arg \inf_{D \in \mathbb{D}} \bar{\sigma} [DS_{\text{nom}}(j\omega)D^{-1}] \quad (3.3)$$

where $\mathbb{D} = \{ \text{diag}(d_1 I, d_2 I, \dots, d_n I) \mid d_j \in \mathbf{R}_+ \}$ (3.4)

$$\gamma_{\text{nom}} = \| D_{\text{nom}} S_{\text{nom}} D_{\text{nom}}^{-1} \|_\infty \quad (3.5)$$

The next step in the CRM is the parameterization of all stabilizing compensators in terms of the free parameter $Q \in H_\infty$ [9, 10]. This parameterization is performed so that the nominal compensator K_{nom} and the nominal closed-loop system S_{nom} result when Q is the zero function [14].

Form the right and left coprime factorizations of the plant transfer function matrix P_{22} .

$$P_{22} = NM^{-1} = \tilde{M}^{-1}\tilde{N} \quad (3.6)$$

It is shown in [14] that functions U and V in RH_∞ may be computed so that

$$-\tilde{N}U + \tilde{M}V = I \quad (3.7)$$

$$K_{\text{nom}} = UV^{-1} \quad (3.8)$$

The following two theorems are well-known.

Theorem 3.1 The set of all proper controllers achieving internal stability for the feedback system in Figure 3.1 is parameterized by the formula

$$K = (U + MQ)(V + NQ)^{-1}, \quad Q \in H_\infty \quad (3.9)$$

The above theorem parameterizes all stabilizing controllers for the plant P in terms of a free parameter Q . The affine parameterization of the stable closed-loop transfer function matrices from exogenous inputs d to errors e follows.

Theorem 3.2 The set of all closed-loop transfer function matrices S from d to e achievable by an internally stabilizing proper controller is

$$\mathcal{S} = \{ S \mid S = T_{11} + T_{12}QT_{21}, Q \in H_\infty, I + D_{22}Q(j\omega) \text{ invertible at } \omega = \infty \} \quad (3.10)$$

$$\text{where } T_{11} = P_{11} + P_{12}U\tilde{M}P_{21}$$

$$= S_{\text{nom}}$$

$$T_{12} = P_{12}M$$

$$T_{21} = \tilde{M}P_{21}$$

Theorem 3.2 parameterizes all stable closed-loop maps from d to e in terms of a stable, causal function Q . The most elementary function in H_∞ is the zero function, and by construction the resulting closed-loop is S_{nom} . However, S_{nom} and the robustness bound γ_{nom} may not represent adequate stability and performance robustness of the feedback system (i.e. $\gamma_{\text{nom}} > 1$). Thus, the aim of the CRM is to *improve* the robustness of the closed-loop system (i.e. decrease the robustness bound γ) by exploiting the extra degree of freedom available in the free parameter Q . The CRM may be thought of as an algorithm to "fine-tune" the nominal design S_{nom} by adjusting the frequency response of the transfer function matrix Q . In the remainder of this section, a procedure is developed to find a $Q \in H_\infty$ such that

$$\| D_{\text{nom}}(T_{11} + T_{12}QT_{21})D_{\text{nom}}^{-1} \|_\infty \leq \gamma_{\text{nom}} \quad (3.11)$$

The implication is clear. Start with a "good" nominal design $T_{11} = S_{\text{nom}}$, and the CRM will produce another closed-loop system whose robustness is at least as good as that of the original design.

Remark The scaling D_{nom} is computed as a function of S_{nom} (Eqn. 3.3), and does not change throughout the CRM process. As we shall see, this greatly simplifies the design problem and leads to a convex program in Q . However, we are now no longer trying to optimize the structured singular value μ ; the infinity norm of the scaled closed-loop system, $D_{\text{nom}}(T_{11} + T_{12}QT_{21})D_{\text{nom}}^{-1}$, will be minimized (for the fixed scaling D_{nom}).

Once a compensator has been computed by the CRM, S_{nom} may be redefined to incorporate this new design. The scaling D_{nom} is recomputed, and the causality recovery process repeated. This represents a different approach to the DK iteration described in [6-8]. As such, the procedure is nonconvex and convergence to the globally optimal compensator and scaling is not guaranteed.

Optimal Noncausal Design

The Causality Recovery Methodology treats the constraints in (3.1) and (3.2) independently. This allows the designer to temporarily ignore the causality restriction on Q and examine the synthesis problem at each frequency. The rationale behind this approach can be simply described in a single-input, single-output context.

A function in H_∞ (i.e. a stable, causal function) is analytic in the right half plane. Cauchy's Integral Theorem applied along the familiar Nyquist contour imposes constraints on the frequency response of such a function (i.e. Bode's gain and phase integral relationships [1]). The phase (gain) of a stable, causal transfer function is completely determined by the gain (phase) over all frequencies. When the stability/causality restriction is lifted, there is no relationship between the gain and phase of a system from one frequency point to the next. Therefore, we can treat each frequency point as independent from every other frequency.

This philosophy allows one to maximize the "robustness" of the feedback system at each frequency using only complex matrix arithmetic. The result is a closed-loop function with "optimal" performance-robustness. In this case, the price paid for such optimality is that the closed-loop system will not be causal in general. That is, the function will be a member of L_∞ , not H_∞ . However, such a system will provide a lower bound on the robustness measure γ .

The frequency by frequency approach to maximizing robustness suggests the following optimization problem for finding the optimal, noncausal function Q^* .

$$Q^*(j\omega) = \arg \min_{Q \in C^{m \times p}} \bar{\sigma} \left\{ D_{\text{nom}}(j\omega) \left[T_{11}(j\omega) + T_{12}(j\omega) Q T_{21}(j\omega) \right] D_{\text{nom}}(j\omega)^{-1} \right\} \quad (3.12)$$

It is easy to prove the following result.

Theorem 3.3 The optimization in (3.12) is a convex program in Q .

Remark The optimization in Eqn. (3.12) is carried out independently at every frequency. The parameters are the real and imaginary parts of the elements of Q . This results in having to calculate 2mp real, scalar parameters at each frequency.

The robustness bound on the optimal noncausal system is

$$\gamma^* = \| D_{\text{nom}}(T_{11} + T_{12}Q^*T_{21})D_{\text{nom}}^{-1} \|_{\infty} \quad (3.13)$$

The measure γ^* is a lower bound on the performance-robustness measure that may be achieved by a $Q \in H_{\infty}$.

Causality Recovery

The optimal noncausal function $Q^* \in L_{\infty}$, and is in general not in H_{∞} . Thus, the restriction imposed by the Youla parameterization is not satisfied and nominal stability of the closed-loop is not achieved by the function Q^* . In this section, we propose an algorithm to find a $Q \in H_{\infty}$ such that the closed-loop performance-robustness is no worse than, and hopefully superior to, that of the nominal design, i.e.

$$\| D_{\text{nom}}(T_{11} + T_{12}QT_{21})D_{\text{nom}}^{-1} \|_{\infty} \leq \gamma_{\text{nom}} \quad (3.14)$$

This process, referred to as causality recovery, may be thought of as an adjustment of the frequency response of Q^* in such a way as to reduce its noncausality, or distance from H_∞ , subject to a robustness constraint on the closed-loop function. An alternative view is that causality recovery is a search for an H_∞ function over a tube in complex matrix space versus frequency. The robust performance specification dictates the geometry of the tube.

Define the feasible set of frequency responses that satisfy the robustness specification γ , for $\gamma^* \leq \gamma \leq \gamma_{\text{nom}}$.

$$\Phi = \{ Q \in L_\infty \mid \| D_{\text{nom}}(T_{11} + T_{12}QT_{21})D_{\text{nom}}^{-1} \|_\infty \leq \gamma \} \quad (3.15)$$

At a specific frequency, the feasible set Φ may be interpreted as a set of complex matrices Q satisfying

$$\overline{\sigma} \{ D_{\text{nom}}(\omega)[T_{11}(j\omega) + T_{12}(j\omega)QT_{21}(j\omega)]D_{\text{nom}}(\omega)^{-1} \} \leq \gamma \quad (3.16)$$

The feasible set Φ contains all L_∞ functions that satisfy the robust performance specification for a given γ . We wish to determine if any of the L_∞ functions in Φ are also in H_∞ . The fundamental component of the Causality Recovery Methodology is an optimization problem to establish the existence of a $Q \in \Phi \cap H_\infty$. Nehari's Theorem [11, 12, 15] states that an L_∞ function Q is in H_∞ if and only if the norm of the Hankel operator with symbol Q , $\| \Gamma_Q \|$, is identically zero. This suggests the following optimization problem.

$$\min_{Q \in \Phi} \| \Gamma_Q \| \quad (3.17)$$

This problem is at the heart of the CRM, and it is easy to prove that:

Theorem 3.4 The optimization in (3.17) is a convex program in Q .

If an H_∞ function lies within Φ (for a given γ), then the minimum in (3.17) is zero and the argument Q results in a nominally stable closed-loop that achieves the robust performance objective. If γ is too small (i.e. the performance specifications are too stringent for the given amount of modeling error), a stable, causal function may not lie in the feasible set and the minimum Hankel norm will be some positive number. A binary search over the interval $[\gamma^*, \gamma_{\text{nom}}]$ can be used to find the minimum γ that admits an H_∞ function into the feasible set. The search procedure is analogous to the γ -iteration that is performed as part of the standard H_∞ design process [11, 12, 13].

The optimization in (3.17) is an infinite-dimensional, convex program due to the definition of Φ as a set of L_∞ functions. For implementation purposes, a finite-dimensional (i.e. computable) algorithm that approximates the optimization program in (3.17) and the CRM γ -iteration has been developed [14]. Unfortunately, convexity is lost in the finite-dimensional case.

Although the Hankel norm optimization is no longer a convex program in Q , the algorithm in [14] guarantees the finding of a finite-dimensional, rational transfer function matrix Q with the following properties.

$$\begin{aligned} (1) \quad & \| \Gamma_Q \| \leq \epsilon \\ (2) \quad & \| D_{\text{nom}}(T_{11} + T_{12}QT_{21})D_{\text{nom}}^{-1} \|_\infty \leq \gamma_{\text{nom}} - k\epsilon \end{aligned}$$

for any $\epsilon > 0$, and some $k > 0$.

A Q_H in H_∞ (i.e. with Hankel norm identically equal to zero) is then computed as the

best H_∞ approximation of the Q produced by the CRM algorithm, using the procedure in [16]. The closed-loop robustness associated with Q_H is within a multiple of ϵ of the robustness measure associated with Q [14]. The CRM compensator K is constructed according to Eqn. (3.9).

4. A NUMERICAL EXAMPLE

This section presents a design example to illustrate feedback system synthesis via the Causality Recovery Methodology. More specifically, we show how the CRM improves the performance-robustness of a feedback system. The problem to be considered is a multivariable system created by Stein [17]. The feedback structure is given by the conventional block diagram in Figure 4.1. There is a multiplicative uncertainty block at the plant input and a performance specification at the plant output. Note that this is a special case of the more general framework in Figure 1.1.

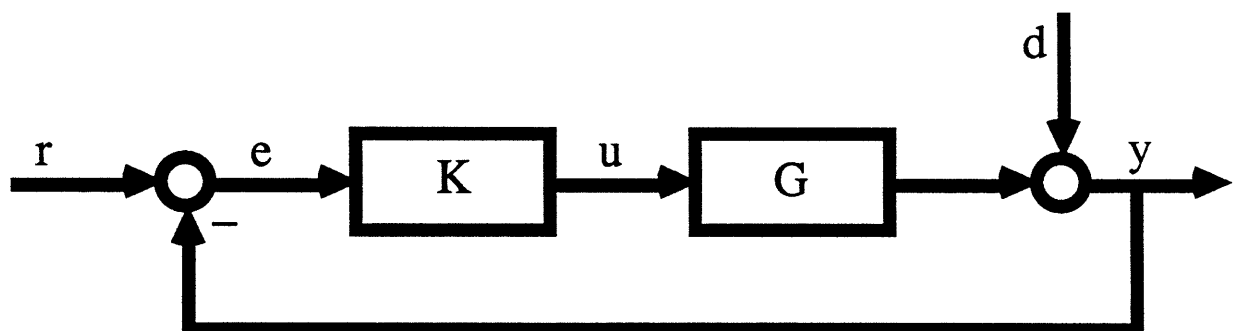


Figure 4.1 Conventional feedback structure.

Given the problem structure in Figure 4.1, the plant P in Figure 1.1 is

$$P = \begin{bmatrix} 0 & 0 & W_z \\ -W_e G & W_e & -W_e G \\ -G & I & -G \end{bmatrix} \quad (4.1)$$

where

G is the plant to be controlled

W_z is the uncertainty weighting function (i.e. the bound on the input multiplicative modeling error)

W_e is the performance weighting function (i.e. the bound on the output sensitivity function)

The nominal plant is

$$G(s) = \frac{1}{s} \begin{bmatrix} a & 0 \\ 0 & 1/a \end{bmatrix} \quad (4.2)$$

For $a = 5$, the singular values of $G(j\omega)$ are shown in Figure 4.2.

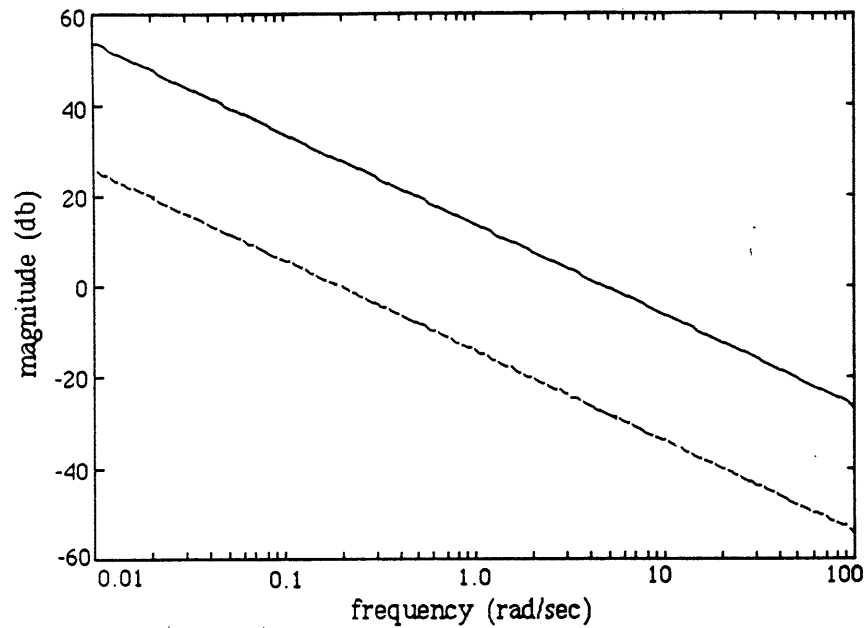


Figure 4.2 The frequency response of the singular values of the nominal plant G .

The multiplicative uncertainty at the plant input results in a perturbed plant

$$\tilde{G} = G[I + L] \quad (4.3)$$

The bound on the multiplicative error L is

$$W_z(s) = 0.5(s + 1) \frac{1000}{s + 1000} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (4.4)$$

The singular values of $W_z(j\omega)$ are shown in Figure 4.3.

After examining Eqns. (4.2) and (4.4), one may conclude that the system is decoupled and can be treated as two SISO problems. This is not the case, however. The diagonal uncertainty weight merely provides a bound on the *singular values* of the multiplicative perturbation; a legal perturbation may be a full transfer function matrix. In such a case, the perturbed plant \tilde{G} would be coupled. *Thus, this problem is truly multivariable in nature and may not be treated as two SISO designs.* In the sequel, we will evaluate the performance of the CRM design with one of these coupled plants.

The performance weighting function was chosen to provide a "cross-over gap" with respect to the uncertainty weight in Eqn. (4.4).

$$W_e(s) = \frac{0.5(s+1)}{s} \frac{1000}{s+1000} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (4.5)$$

The singular values of $W_e(j\omega)$ are shown in Figure 4.3.

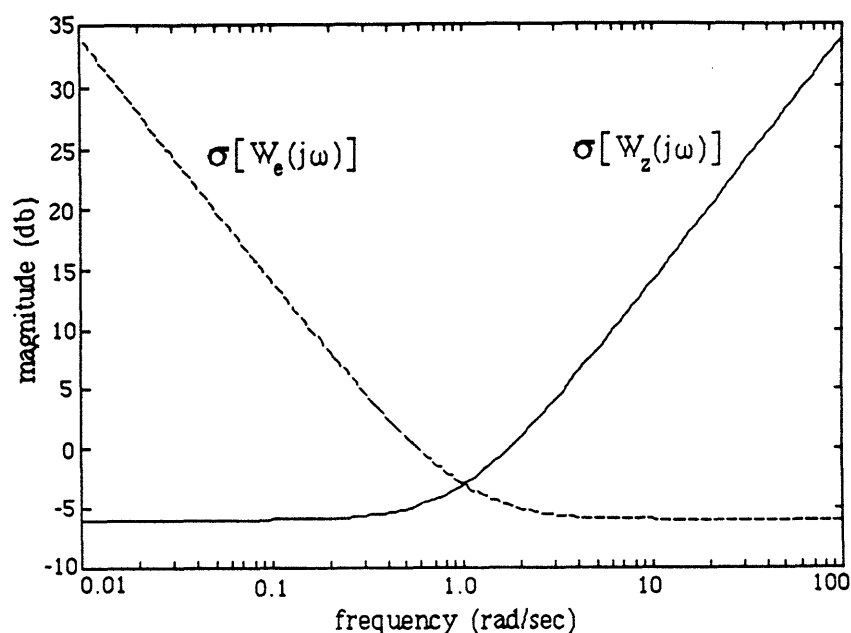


Figure 4.3 The singular values of the uncertainty and performance weighting functions.

A (four-block) H_∞ design was performed for the plant model P in Eqn. (4.1). A recent advance by Doyle and Glover [18] allows one to efficiently solve H_∞ feedback problems through the solution of two Riccati equations. This procedure was used to compute the diagonal H_∞ compensator K_{nom} shown in Figure 4.4. The characteristics of the closed-loop transfer function $S_{\text{nom}} = F_l(P, K_{\text{nom}})$ are plotted in Figure 4.5. The robustness bound of the H_∞ design is $\gamma_{\text{nom}} = 1.91$.

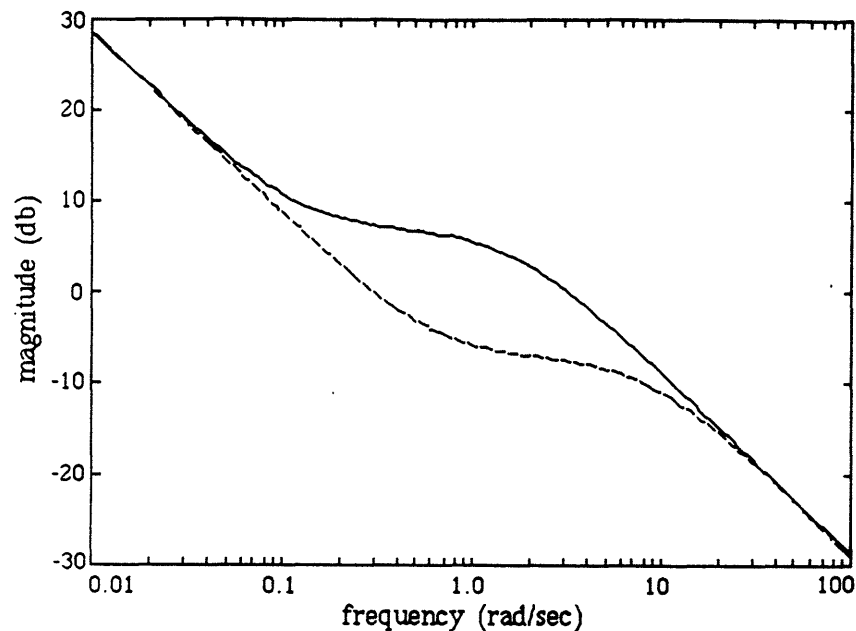


Figure 4.4 The singular values of the H_∞ compensator K_{nom} .

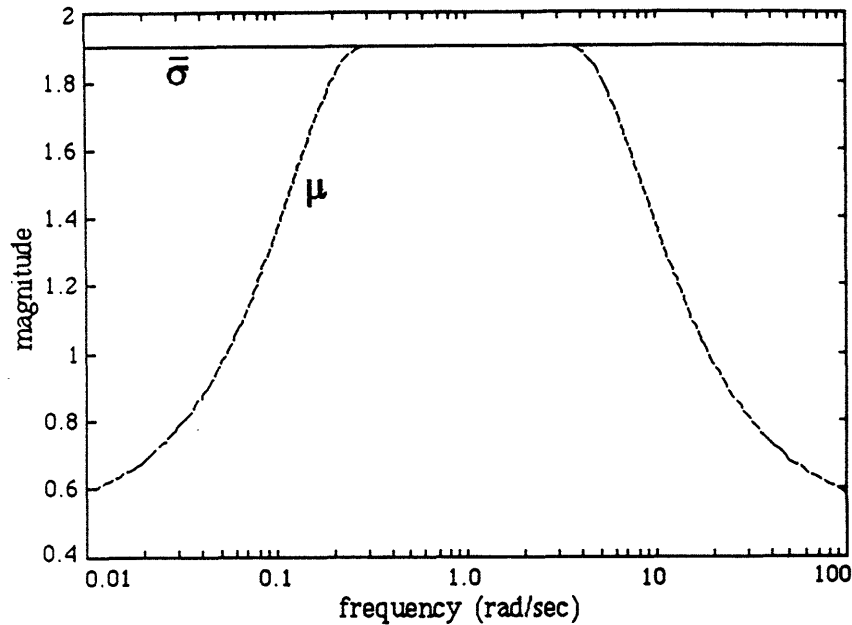


Figure 4.5 Characteristics of the closed-loop transfer function S_{nom} for the H_{∞} design.

The CRM procedure was carried out as described in Section 3, and the singular values of the resulting diagonal compensator are shown in Figure 4.6. The frequency response of the CRM scaled closed-loop transfer function matrix, $D_{\text{nom}}(T_{11} + T_{12}QT_{21})D_{\text{nom}}^{-1}$, is shown in Figure 4.7. The robustness bound in this case has been reduced to 1.61 (compare with the H_{∞} design $\gamma_{\text{nom}} = 1.91$). Thus, the CRM has improved the performance-robustness of the closed-loop system.

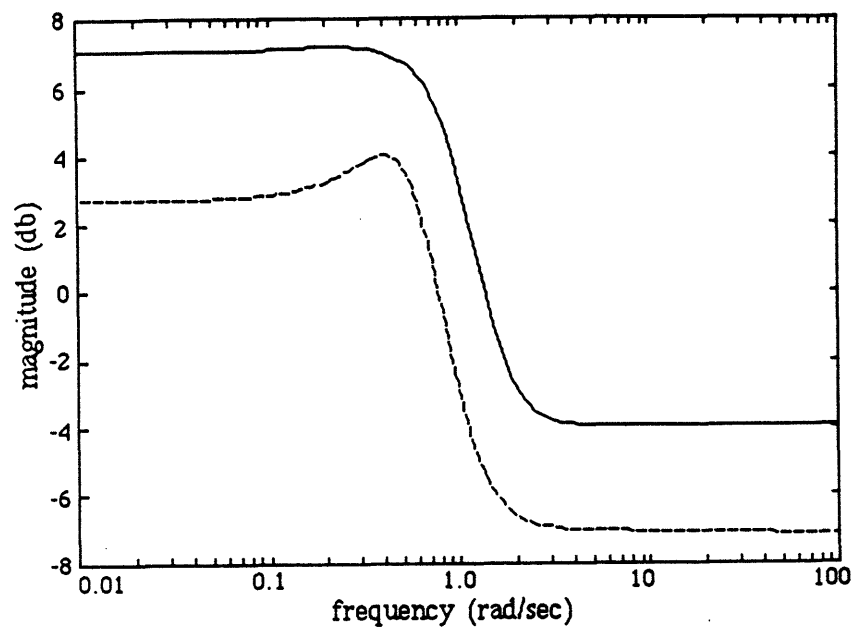


Figure 4.6 The frequency response of the singular values of the CRM compensator.

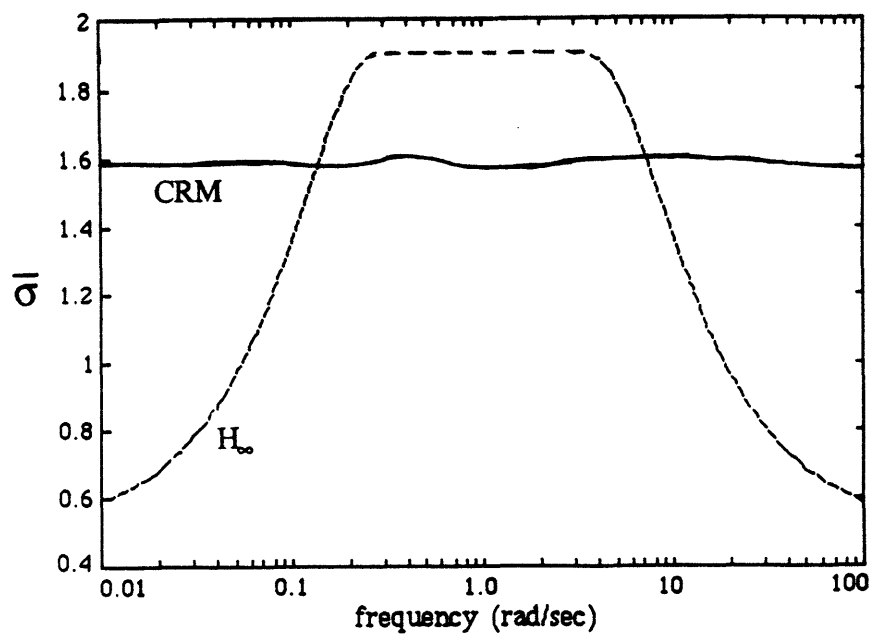


Figure 4.7 The response of the largest singular value of the CRM closed-loop matrix.

The implications of reducing the robustness bound are best understood in the context of the conventional feedback loop in Figure 4.1. A performance comparison between the H_∞ and CRM compensators will be made for a given reference command.

The output (y) responses to a reference command $r = [1 \ 1]'$ are shown in Figures 4.8 and 4.9. The y_1 response of the H_∞ design exhibits 18% overshoot and no undershoot (Figure 4.8). The y_1 response of the CRM design has the same overshoot, and a little undershoot (Figure 4.9). However, the settling times of the two designs are approximately the same (6 seconds). The y_2 CRM response has much less overshoot and a significantly faster settling time when compared to the H_∞ design.

The true benefits of a robust design methodology, such as the CRM, are brought to light when the plant in the feedback loop is other than the nominal plant G . From Eqn. (4.3), a perturbed plant \tilde{G} is a product of the nominal plant G and some multiplicative input uncertainty. The following transfer function matrix is a *legal* plant, as defined by the set of admissible perturbations Δ and the uncertainty weight $W_z(s)$.

$$\tilde{G} = \begin{bmatrix} \frac{a}{s} & \frac{ka}{s+5} \\ \frac{ka^{-1}}{s+5} & \frac{a^{-1}}{s} \end{bmatrix} \quad (4.6)$$

where $a = 5$ and $k = 1.75$. The response of the system in Figure 4.1 is examined for the case when the perturbed plant \tilde{G} is in the feedback loop.

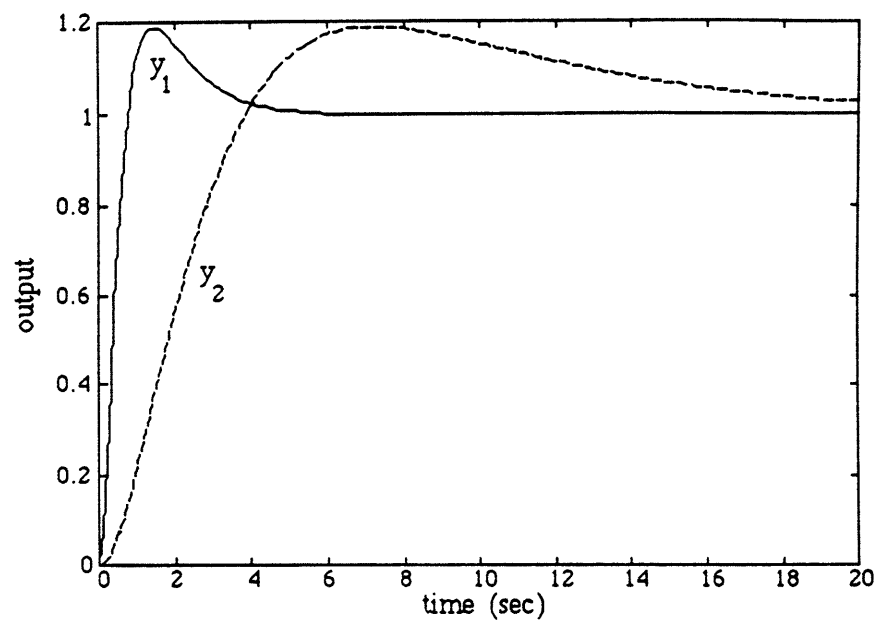


Figure 4.8 The closed-loop output response to an input command $r = [1 \ 1]'$ with plant G and the H_∞ compensator.

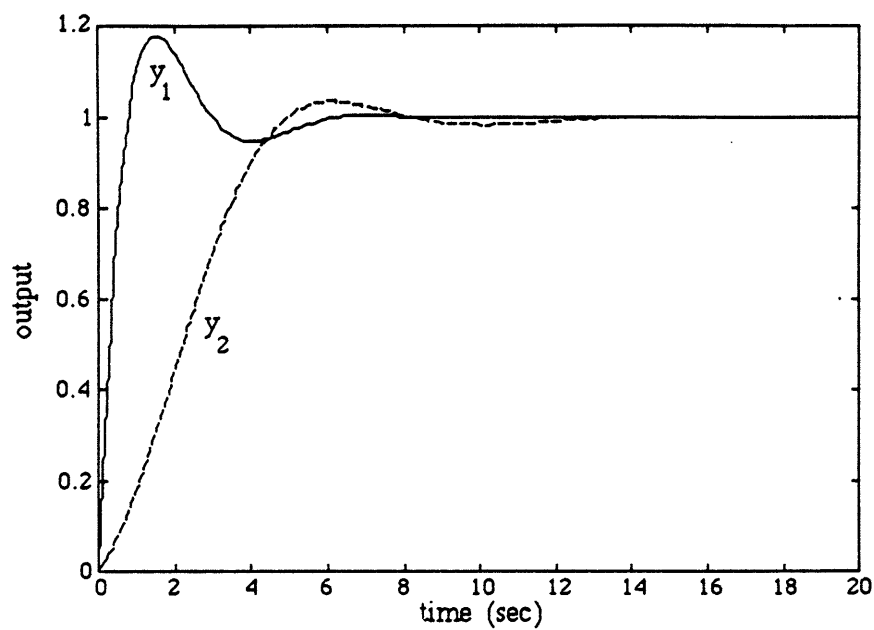


Figure 4.9 The closed-loop output response to an input command $r = [1 \ 1]'$ with plant G and the CRM compensator.

The H_∞ compensator and the perturbed plant produce a poor y_1 step response, shown in Figure 4.10. However, the y_2 response is virtually unaffected by the perturbation. The response of the CRM design, with \tilde{G} in the loop, is shown in Figure 4.11. The y_1 response exhibits more than twice the overshoot, when compared to the response with G in the loop, but this is significantly better than the H_∞ design. Note that the y_2 response is largely unaffected by the perturbation because of the factor of a^{-1} (0.2) in the \tilde{G}_{21} transfer function (Eqn. 4.6).

The CRM design objective of increasing the performance-robustness of the feedback loop was achieved. This resulted in better closed-loop performance, particularly when a plant other than the nominal was in the feedback loop. That is, the degradation in feedback performance resulting from plant perturbations was much less severe for the CRM design than for the H_∞ compensator. This suggests that the four-block H_∞ design is not particularly well-suited for handling significant amounts of plant modeling error, at least in this simple example.

The most significant drawback of the CRM is the computational inefficiency of the causality recovery algorithm, as a consequence of the huge number of optimization problems being solved. Several days of computation were required on a Micro-VAX workstation. Clearly, the severe computational burden is sufficient to make the CRM impractical at this time if implemented on a serial machine. However, the optimization programs should be parallelizable for super-computer implementation. Nonetheless, in view of the recent breakthrough in efficiently solving H_∞ feedback problems [18], Doyle's DK iteration method [6-8] requires more modest resources to converge to a (local) minimum.

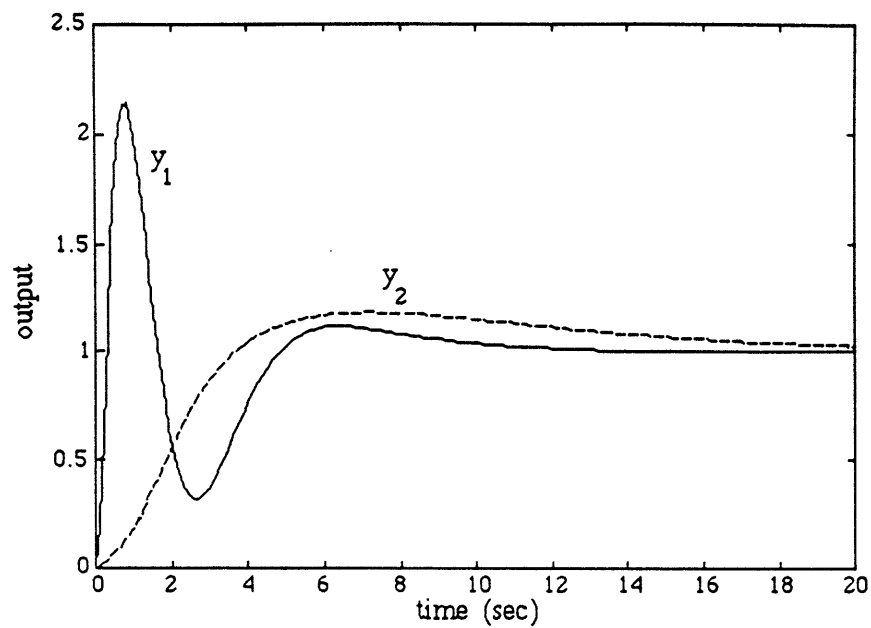


Figure 4.10 The closed-loop output response to an input command $r = [1 \ 1]'$ with perturbed plant \tilde{G} and the H_∞ compensator.

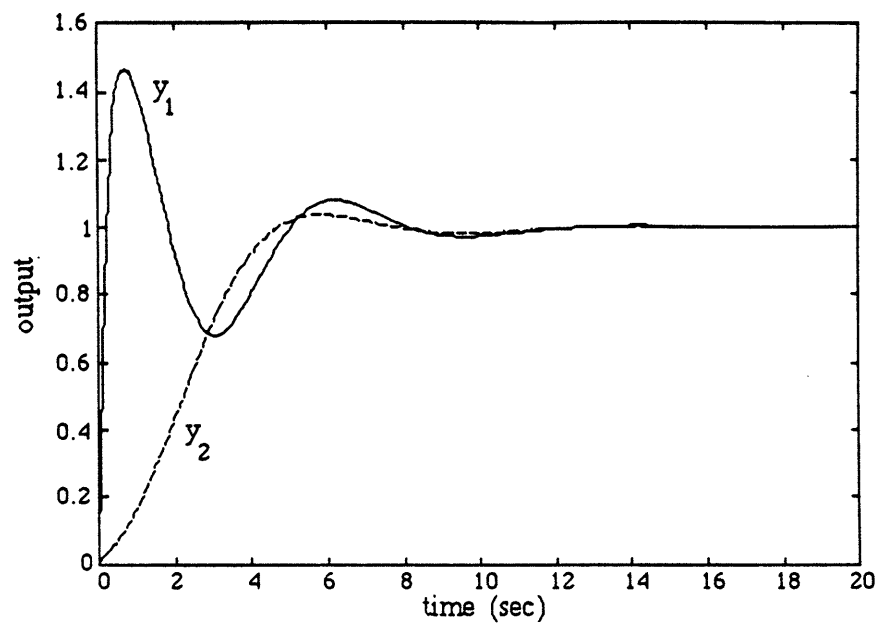


Figure 4.11 The closed-loop output response to an input command $r = [1 \ 1]'$ with perturbed plant \tilde{G} and the CRM compensator.

5. CONCLUSIONS

A new design technique, the Causality Recovery Methodology, has been developed for the synthesis of finite-dimensional, linear, time-invariant feedback systems. Stability *and* performance in the presence of multiple, unstructured modeling uncertainties is guaranteed. The CRM will produce a closed-loop system whose performance-robustness, expressed in terms of the structured singular value, is better than or equal to that of a given (nominal) feedback system. Thus, the CRM may be used as a stepping stone for a new DK iteration for robust synthesis.

The numerical example demonstrates that the CRM is a viable design concept. While these preliminary results are encouraging, the tremendous computational cost associated with the robustness enhancement makes the method impractical for implementation on serial machines at this time.

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